

NUMBER SYSTEMS

We are familiar with the following sets of numbers

i) N = Set of natural numbers
= $\{1, 2, 3, \dots\}$ (Counting Set)

ii) W = Set of whole numbers
= $\{0, 1, 2, \dots\}$

iii) Z = Set of all integers
= $\{0, \pm 1, \pm 2, \dots\}$

iv) Q = Set of Rational numbers
= $\{\frac{p}{q} / p, q \in Z \text{ and } q \neq 0\}$
(\wedge stands for "and")

v) Q' = Set of irrational numbers
= $\{\pm \sqrt{a} / a \text{ is not a perfect square}\}$

vi) R = Set of all Real numbers
= $Q \cup Q'$

Decimal Representation of Rational and Irrational numbers.

1. Terminating Decimals

A decimal which has only a finite number of decimal places, is called a Terminating Decimal. e.g. 202.04, 0.00415, 1000, 1236 etc.

Since every terminating decimal can be converted into a common fraction, therefore it is a rational number.

2. Recurring or Periodic Decimals

A decimal in which one or more digits repeat indefinitely, is called a recurring or periodic decimal. e.g. $1.333\dots = 1.\bar{3}$, $0.818181\dots = 0.\bar{81}$.

Since every recurring decimal can be converted into a common fraction, therefore it is a rational number.

3. Non-Terminating, Non-Recurring Decimals

A decimal which neither terminates nor it is recurring is called non-terminating, non-recurring decimal.

A decimal of such type is an irrational number.

EXAMPLE

i) $0.25 (= \frac{25}{100})$ is a rational number.
(Terminating decimal)

ii) $0.333\dots (= 0.\bar{3}) (= \frac{1}{3})$ is a rational number (Recurring decimal)

iii) $2.333\dots (= 2.\bar{3})$ is a rational number (Recurring decimal)

iv) $0.412857142857\dots (= 0.\overline{412857})$ is a rational no. (Recurring decimal)

v) $0.0100100010000\dots$ is an irrational number (non-terminating, non-recurring)

vi) $2.1412112211122211112222\dots$ is also an irrational number. (non-terminating, non-recurring)

vii) 3.141592654... is an irrational number. (non-terminating, non-recurring) called π which denotes the constant ratio of the circumference of any circle to the length of its diameter. i.e;

$$\pi = \frac{\text{Circumference of any circle}}{\text{length of its diameter}}$$

An approximate value of π is $\frac{22}{7}$, a better approximation is

$$\frac{355}{113} \text{ and still a better}$$

approximation is 3.14159. All of these are rational numbers.

These approximations make the calculation of problems involving π simpler and easier.

EXAMPLE

Prove $\sqrt{2}$ is an irrational number.

Sol: Suppose $\sqrt{2}$ is rational, then $\sqrt{2} = \frac{p}{q}$ where $p, q \in \mathbb{Z}$, $q \neq 0$ and suppose that p and q have no common divisor other than 1.

$$\therefore \sqrt{2} = \frac{p}{q}, \quad q \neq 0$$

Squaring both sides, we get

$$2 = \frac{p^2}{q^2}$$

$$\Rightarrow 2q^2 = p^2 \text{ or } p^2 = 2q^2$$

$$\Rightarrow p^2 \text{ is divisible by } 2$$

$$\Rightarrow p \text{ is divisible by } 2$$

$$\Rightarrow p^2 \text{ is divisible by } 4$$

$$\Rightarrow 2q^2 \text{ is divisible by } 4$$

$$\Rightarrow q^2 \text{ is divisible by } 2$$

$$\Rightarrow q \text{ is divisible by } 2$$

\Rightarrow both p & q are divisible by 2

\Rightarrow both p & q have a common divisor 2 and $2 \neq 1$

\Rightarrow a contradiction

Thus our supposition is wrong and hence $\sqrt{2}$ is an irrational number.

EXAMPLE

Prove $\sqrt{3}$ is an irrational number.

Solution:

Suppose $\sqrt{3}$ is rational, then

$$\sqrt{3} = \frac{p}{q} \text{ where } p, q \in \mathbb{Z}, \quad q \neq 0$$

and suppose that p & q have no common divisor other than 1.

$$\therefore \sqrt{3} = \frac{p}{q}$$

Squaring both sides, we get

$$3 = \frac{p^2}{q^2}$$

$$\Rightarrow 3q^2 = p^2 \text{ or } p^2 = 3q^2$$

$$\Rightarrow p^2 \text{ is divisible by } 3$$

$$\Rightarrow p \text{ is divisible by } 3$$

$$\Rightarrow p^2 \text{ is divisible by } 9$$

$$\Rightarrow 3q^2 \text{ is divisible by } 9$$

$$\Rightarrow q^2 \text{ is divisible by } 3$$

$$\Rightarrow q \text{ is divisible by } 3$$

$$\Rightarrow \text{both } p \text{ & } q \text{ are divisible by } 3.$$

$$\Rightarrow \text{both } p \text{ & } q \text{ have a common divisor } 3 \text{ and } 3 \neq 1$$

$$\Rightarrow \text{a contradiction}$$

Thus our supposition is wrong

and hence $\sqrt{3}$ is an irrational number.

Properties of Real Numbers:

Binary Operation

A binary operation in a set S is a rule denoted by $*$ that assigns to any pair of elements of S a unique element of S . i.e., for $a, b \in S$, $*$ is a binary operation if $*(a, b) = a * b \in S$.
Following are the properties or laws of real numbers.

1. Addition Laws

i) Closure Law of Addition

$$\forall a, b \in \mathbb{R}, a + b \in \mathbb{R}$$

ii) Associative Law

$$\forall a, b, c \in \mathbb{R}$$

$$a + (b + c) = (a + b) + c$$

iii) Additive Identity

$$\forall a \in \mathbb{R}, \exists 0 \in \mathbb{R} \text{ such that}$$

$$a + 0 = 0 + a = a$$

0 is called additive identity.

iv) Additive Inverse

$$\forall a \in \mathbb{R} \exists -a \in \mathbb{R} \text{ such that}$$

$$a + (-a) = 0 = -a + a$$

a and $-a$ are called additive inverse of each other.

v) Commutative Law

$$\forall a, b \in \mathbb{R}$$

$$a + b = b + a$$

2. Multiplication Laws

vi) Closure Law

$$\forall a, b \in \mathbb{R}, a \cdot b \in \mathbb{R}$$

vii) Associative Law

$$\forall a, b, c \in \mathbb{R}$$

$$a(bc) = (ab)c$$

viii) Multiplicative Identity

Identity

$$\forall a \in \mathbb{R}, \exists 1 \in \mathbb{R} \text{ such that}$$

$$a \cdot 1 = 1 \cdot a = a \cdot 1 \text{ is called multiplicative identity}$$

ix) Multiplicative Inverse

Inverse

$$\forall a \in \mathbb{R}, a \neq 0 \exists a^{-1} \in \mathbb{R} \text{ such that}$$

$$a \cdot a^{-1} = 1 = a^{-1} \cdot a \text{ where}$$

$$a^{-1} = \frac{1}{a}$$

a and a^{-1} are called multiplicative inverse of each other.

x) Commutative Law

$$\forall a, b \in \mathbb{R}, ab = ba$$

3. Multiplication-Addition Law

$$xi) \forall a, b, c \in \mathbb{R}$$

$$a \cdot (b + c) = ab + ac \quad \left(\begin{array}{l} \text{left} \\ \text{distributive law} \end{array} \right)$$

and

$$(a + b) \cdot c = ac + bc \quad \left(\begin{array}{l} \text{Right distrib.} \\ \text{law} \end{array} \right)$$

4. Properties of Equality

i) Reflexive Property

$$\forall a \in \mathbb{R}, a = a$$

ii) Symmetric Property

$$\forall a, b \in \mathbb{R} \quad a = b \Rightarrow b = a$$

iii) Transitive Property

$$\forall a, b, c \in \mathbb{R}$$

$$a = b \wedge b = c \Rightarrow a = c$$

iv) Additive Property

$$\forall a, b, c \in \mathbb{R}$$

$$a = b \Rightarrow a + c = b + c$$

v) Multiplicative Property

$$\forall a, b, c \in \mathbb{R}$$

$$a = b \Rightarrow ac = bc \wedge ca = cb$$

vi) Cancellation Law w.r.t. '+'

$$\forall a, b, c \in \mathbb{R} \quad a + c = b + c \Rightarrow a = b$$

vii) Cancellation Law w.r.t. '·'

$$\forall a, b, c \in \mathbb{R}, ac = bc \Rightarrow a = b, c \neq 0$$

5. Properties of Inequalities (Order Properties)

i) Trichotomy Property

$$\forall a, b \in \mathbb{R}$$

either $a = b$ or $a < b$ or $a > b$

ii) Transitive Property

$$\forall a, b, c \in \mathbb{R}$$

$$a > b \wedge b > c \Rightarrow a > c$$

$$a < b \wedge b < c \Rightarrow a < c$$

iii) Additive Property

$$\forall a, b, c \in \mathbb{R}$$

$$a > b \Rightarrow a + c > b + c \text{ and}$$

$$a < b \Rightarrow a + c < b + c$$

$$b) a > b \wedge c > d \Rightarrow a + c > b + d$$

$$\text{and } a < b \wedge c < d \Rightarrow a + c < b + d$$

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iv) Multiplicative Property

$$a) \forall a, b, c \in \mathbb{R} \text{ and } c > 0$$

$$a > b \Rightarrow ac > bc \text{ and}$$

$$a < b \Rightarrow ac < bc$$

$$b) \forall a, b, c \in \mathbb{R} \text{ and } c < 0$$

$$a > b \Rightarrow ac < bc \text{ and}$$

$$a < b \Rightarrow ac > bc$$

$$c) \forall a, b, c, d \in \mathbb{R} \text{ (all +ve)}$$

$$a > b \wedge c > d \Rightarrow ac > bd \text{ and}$$

$$a < b \wedge c < d \Rightarrow ac < bd$$

EXAMPLE 4

Prove that for any real numbers a, b

$$i) a \cdot 0 = 0$$

$$ii) ab = 0 \Rightarrow a = 0 \vee b = 0$$

Proof

$$i) a \cdot 0 = a [1 + (-1)] \text{ (additive inverse)}$$

$$= a(1 - 1) \text{ (Def. of subtraction)}$$

$$= a \cdot 1 - a \cdot 1 \text{ (Distributive Law)}$$

$$= a - a \text{ (1 is multiplicative identity)}$$

$$= a + (-a) \text{ (Def. of sub.)}$$

$$a \cdot 0 = 0 \text{ (Additive inverse)}$$

ii) Given that

$$ab = 0 \text{ ——— ①}$$

Suppose $a \neq 0$ then $\frac{1}{a}$ exists

From ①

$$\frac{1}{a}(ab) = \frac{1}{a} \cdot 0 \text{ (multiplicative Property)}$$

$$\frac{1}{a}(ab) = 0 \therefore a \cdot 0 = 0$$

$$(\frac{1}{a} \cdot a)b = 0 \text{ (Assoc. Law of '·')}$$

$$1 \cdot b = 0 \text{ (multiplicative inverse)}$$

$$b = 0 \text{ (1 is multiplicative identity)}$$

Thus if $ab = 0$, $a \neq 0$ then $b = 0$

Similarly it can be proved that if $ab=0$ and $b \neq 0$ then $a=0$.

Hence $ab=0 \Rightarrow a=0$ or $b=0$

EXAMPLE 5.

For real numbers a, b show the following by stating the properties used.

i) $(-a)b = a(-b) = -ab$

ii) $(-a)(-b) = ab$

Proof

i) $(-a)b + ab = (-a+a)b$ (dist. law)
 $= 0 \cdot b$ (additive inverse)
 $= 0$

$\Rightarrow (-a)b$ and ab are additive inverse of each other.

$\therefore (-a)b = -(ab) = -ab$

Now

$a(-b) + ab = a(-b+b)$ (Distributive law)
 $= a \cdot 0$ (additive inverse)
 $= 0$

$\Rightarrow a(-b)$ and ab are additive inverse of each other.

$\therefore a(-b) = -(ab) = -ab$

ii) $(-a)(-b) + [-(ab)]$

$= (-a)(-b) + (-ab)$

$= (-a)(-b) + (-a)(b)$ (By i)

$= (-a)(-b+b)$ (Distributive law)

$= (-a) \cdot 0 = 0$

$\Rightarrow (-a)(-b)$ and $-(ab)$ are additive inverse of each other.

EXAMPLE 6.

Sol: (i) Suppose $\frac{a}{b} = \frac{c}{d}$
 $\frac{a}{b} \times 1 = 1 \times \frac{c}{d}$ (multiplicative identity)

$\frac{a}{b} \times (d \times \frac{1}{d}) = (b \times \frac{1}{b}) \times \frac{c}{d}$ (mult. inverse)

$\frac{a}{b} \times \frac{d}{d} = \frac{b}{b} \times \frac{c}{d}$ ($\because \frac{a}{b} = a \times \frac{1}{b}$)

$\frac{ad}{bd} = \frac{bc}{bd}$

$ad \times \frac{1}{bd} = bc \times \frac{1}{bd}$ ($\because \frac{a}{b} = a \times \frac{1}{b}$)

$ad = bc$ (cancellation law)

Conversely suppose that

$ad = bc$

$(ad) \times (\frac{1}{bd}) = (bc) \times (\frac{1}{bd})$ [mult. property]

$(ad) \times (\frac{1}{b} \times \frac{1}{d}) = (bc) \times (\frac{1}{b} \times \frac{1}{d})$

$(ad) \times (\frac{1}{d} \times \frac{1}{b}) = (cb) \times (\frac{1}{b} \times \frac{1}{d})$
 [closure Law]

$a \cdot (d \cdot \frac{1}{d}) \cdot \frac{1}{b} = c \cdot (b \cdot \frac{1}{b}) \cdot \frac{1}{d}$
 (Associative Law)

$a \cdot 1 \cdot \frac{1}{b} = c \cdot 1 \cdot \frac{1}{d}$ [mult. inverse]

$a \cdot \frac{1}{b} = c \cdot \frac{1}{d}$ [mult. identity]

$\frac{a}{b} = \frac{c}{d}$ (proved)

To prove-

$$ii) \frac{1}{a} \cdot \frac{1}{b} = \frac{1}{ab}$$

consider

$$\begin{aligned} (ab) \left(\frac{1}{a} \cdot \frac{1}{b} \right) &= (ba) \left(\frac{1}{a} \cdot \frac{1}{b} \right) \text{ [closure law]} \\ &= b \cdot \left(a \cdot \frac{1}{a} \right) \cdot \frac{1}{b} \text{ (Assoc. Law)} \\ &= b \cdot 1 \cdot \frac{1}{b} = b \cdot \frac{1}{b} = 1 \end{aligned}$$

$\Rightarrow ab$ and $\frac{1}{a} \cdot \frac{1}{b}$ are multiplicative inverse of each other.

$$\therefore \frac{1}{a} \cdot \frac{1}{b} = \frac{1}{ab}$$

$$iii) \text{ To prove } \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

$$\text{L.H.S.} = \frac{a}{b} \cdot \frac{c}{d}$$

$$= \left(a \cdot \frac{1}{b} \right) \cdot \left(c \cdot \frac{1}{d} \right)$$

$$= a \cdot \left(\frac{1}{b} \cdot c \right) \cdot \frac{1}{d} \text{ (Assoc. Law)}$$

$$= a \cdot \left(c \cdot \frac{1}{b} \right) \cdot \frac{1}{d} \text{ (Commutative Law)}$$

$$= (ac) \left(\frac{1}{b} \cdot \frac{1}{d} \right) \text{ (Assoc. Law)}$$

$$= ac \cdot \frac{1}{bd}$$

$$= \frac{ac}{bd} = \text{R.H.S.}$$

$$iv) \text{ To prove } \frac{a}{b} = \frac{ka}{kb} \quad (k \neq 0)$$

$$\text{L.H.S.} = \frac{a}{b}$$

$$= \frac{a}{b} \cdot 1 \text{ (multiplicative identity)}$$

$$= \frac{a}{b} \cdot \left(k \cdot \frac{1}{k} \right) \text{ (multiplicative inverse)}$$

$$= \frac{a}{b} \cdot \frac{k}{k}$$

$$= \frac{ak}{bk} = \text{R.H.S.}$$

To prove

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}$$

$$\boxed{6} \quad \text{L.H.S.} = \frac{\frac{a}{b}}{\frac{c}{d}}$$

$$= \frac{\frac{a}{b} \times 1}{1 \times \frac{c}{d}} \text{ [Multiplicative Identity]}$$

$$= \frac{\frac{a}{b} \times \left(d \times \frac{1}{d} \right)}{\left(b \times \frac{1}{b} \right) \times \frac{c}{d}} \text{ [Multiplicative Inverse]}$$

$$= \frac{\frac{a}{b} \times \frac{d}{d}}{\frac{b}{b} \times \frac{c}{d}} = \frac{\frac{ad}{bd}}{\frac{bc}{bd}}$$

$$= \frac{ad \times \frac{1}{bd}}{bc \times \frac{1}{bd}}$$

$$= \frac{ad}{bc} \text{ [Cancellation Law]}$$

$$= \text{R.H.S.}$$

* EXERCISE 1.1 *

① Which of the following sets have closure property w.r.t. '+' and 'x'?

i) $\{0\}$

Addition Table

| | |
|---|---|
| + | 0 |
| 0 | 0 |

$\because 0+0=0 \in \{0\} \Rightarrow \{0\}$ has closure property w.r.t. '+'

Multiplication Table

| | |
|---|---|
| x | 0 |
| 0 | 0 |

$\because 0 \times 0 = 0 \in \{0\} \Rightarrow \{0\}$ has closure property w.r.t. 'x'.

ii) $\{1\}$

Addition Table

| | |
|---|---|
| + | 1 |
| 1 | 2 |

$\because 1+1=2 \notin \{1\} \Rightarrow \{1\}$ does not have closure property w.r.t. '+'

Multiplication Table

| \times | 1 |
|----------|---|
| 1 | 1 |

$\because 1 \times 1 = 1 \in \{1\} \Rightarrow \{1\}$ has closure property w.r.t. ' \times '

iii) $\{0, -1\}$

Addition Table

| + | 0 | -1 |
|----|----|----|
| 0 | 0 | -1 |
| -1 | -1 | -2 |

$\because 0 + 0 = 0 \in \{0, -1\}$

$0 + (-1) = -1 \in \{0, -1\}$

$(-1) + 0 = -1 \in \{0, -1\}$

$(-1) + (-1) = -2 \notin \{0, -1\}$

$\Rightarrow \{0, -1\}$ does not have closure property w.r.t. ' $+$ '

Multiplication Table

| \times | 0 | -1 |
|----------|---|----|
| 0 | 0 | 0 |
| -1 | 0 | 1 |

$\because 0 \times 0 = 0 \in \{0, -1\}$

$0 \times (-1) = 0 \in \{0, -1\}$

$(-1) \times 0 = 0 \in \{0, -1\}$

$(-1) \times (-1) = 1 \notin \{0, -1\}$

$\Rightarrow \{0, -1\}$ does not have closure property w.r.t. ' \times '

iv) $\{1, -1\}$

Addition Table

| + | 1 | -1 |
|----|---|----|
| 1 | 2 | 0 |
| -1 | 0 | -2 |

$\because 1 + 1 = 2 \notin \{1, -1\}$

$1 + (-1) = 0 \notin \{1, -1\}$

$(-1) + 1 = 0 \notin \{1, -1\}$

$(-1) + (-1) = -2 \notin \{1, -1\}$

$\Rightarrow \{1, -1\}$ does not have closure property w.r.t. ' $+$ '

Multiplication Table

| \times | 1 | -1 |
|----------|----|----|
| 1 | 1 | -1 |
| -1 | -1 | 1 |

$\because 1 \times 1 = 1 \in \{1, -1\}$

$1 \times (-1) = -1 \in \{1, -1\}$

$(-1) \times 1 = -1 \in \{1, -1\}$

$(-1) \times (-1) = 1 \in \{1, -1\}$

$\Rightarrow \{1, -1\}$ has closure property w.r.t. ' \times '

② Name the properties used in the following questions.

i) $4 + 9 = 9 + 4$ (Commutative property w.r.t. ' $+$ ')

ii) $(a + 1) + \frac{3}{4} = a + (1 + \frac{3}{4})$ (Assoc. property w.r.t. ' $+$ ')

iii) $(\sqrt{3} + \sqrt{5}) + \sqrt{7} = \sqrt{3} + (\sqrt{5} + \sqrt{7})$ ()

iv) $100 + 0 = 100$ (Additive Identity)

v) $1000 \times 1 = 1000$ (Multiplicative Identity)

vi) $4 + (-4) = 0$ (Additive inverse)

vii) $a - a = 0$ (Additive inverse)

viii) $\sqrt{2} \times \sqrt{5} = \sqrt{5} \times \sqrt{2}$ (Commutative property w.r.t. ' \times ')

ix) $a(b - c) = ab - ac$ (Left distributive property)

x) $(x - y)z = xz - yz$ (Right distributive property)

xi) $4 \times (5 \times 8) = (4 \times 5) \times 8$ (Associative property w.r.t. ' \times ')

xii) $a(b + c - d) = ab + ac - ad$ (Left distributive property)

③ Name the properties used in the following inequalities.

i) $-3 < -2 \Rightarrow 0 < 1$ (Additive property)

ii) $-5 < -4 \Rightarrow 2 > 16$ (multiplicative property)

iii) $1 > -1 \Rightarrow -3 > -5$ (Additive property)

iv) $a < 0 \Rightarrow -a > 0$ (multiplicative property)

v) $a > b \Rightarrow \frac{1}{a} < \frac{1}{b}$ ()

vi) $a > b \Rightarrow -a < -b$ ()

④ Prove the following rules of addition.

i) $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$

L.H.S. = $\frac{a}{c} + \frac{b}{c}$

= $a \times \frac{1}{c} + b \times \frac{1}{c}$

= $(a+b) \times \frac{1}{c}$ (Right dist. property)

= $\frac{a+b}{c} = \text{R.H.S.}$

ii) $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$

L.H.S. = $\frac{a}{b} + \frac{c}{d}$

= $\frac{a}{b} \times 1 + 1 \times \frac{c}{d}$

= $\frac{a}{b} \times (d \times \frac{1}{d}) + (b \times \frac{1}{b}) \times \frac{c}{d}$

= $\frac{a}{b} \times \frac{d}{d} + \frac{b}{b} \times \frac{c}{d}$

= $\frac{ad}{bd} + \frac{bc}{bd}$

= $ad \times \frac{1}{bd} + bc \times \frac{1}{bd}$

= $(ad+bc) \times \frac{1}{bd}$

= $\frac{ad+bc}{bd} = \text{R.H.S.}$

⑤ Prove that

$$-\frac{7}{12} - \frac{5}{18} = \frac{-21-10}{36}$$

L.H.S. = $-\frac{7}{12} - \frac{5}{18}$

= $-\frac{7}{12} \times 1 - \frac{5}{18} \times 1$

= $-\frac{7}{12} \times (3 \times \frac{1}{3}) - \frac{5}{18} \times (2 \times \frac{1}{2})$

= $-\frac{7}{12} \times \frac{3}{3} - \frac{5}{18} \times \frac{2}{2}$

= $-\frac{21}{36} - \frac{10}{36}$

= $-21 \times \frac{1}{36} - 10 \times \frac{1}{36}$

= $(-21-10) \times \frac{1}{36}$

= $\frac{-21-10}{36} = \text{R.H.S.}$

⑥ Simplify by justifying each step.

i) $\frac{4+16x}{4} = \frac{1}{4} \times (4+16x) \because \frac{a}{b} = \frac{1}{b} \times a$

= $\frac{1}{4} \times (4x + 4 \times 4x)$ (multiplicative Identity)

= $\frac{1}{4} \times 4x(1+4x)$ (Distributive Property)

= $1 \times (1+4x)$ (multiplicative inverse)

= $1+4x$ (multiplicative Identity)

ii) $\frac{1}{4} + \frac{1}{5} = \frac{1}{4} \times 1 + \frac{1}{5} \times 1$ (mult. identity)

= $\frac{1}{4} \times (5 \times \frac{1}{5}) + \frac{1}{5} \times (4 \times \frac{1}{4})$ (mult. inverse)

= $\frac{1}{4} \times \frac{5}{5} + \frac{1}{5} \times \frac{4}{4} \because \frac{a}{b} = a \times \frac{1}{b}$

= $\frac{5}{20} + \frac{4}{20}$

= $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

$$\begin{aligned}
 &= \frac{5 \times \frac{1}{20} + 4 \times \frac{1}{20}}{5 \times \frac{1}{20} - 4 \times \frac{1}{20}} \quad \because \frac{a}{b} = a \times \frac{1}{b} \\
 &= \frac{(5+4) \times \frac{1}{20}}{(5-4) \times \frac{1}{20}} \quad (\text{Dist. property}) \\
 &= \frac{5+4}{5-4} \quad (\text{Cancellation law}) \\
 &= \frac{9}{1} = 9
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{\frac{1}{a} \times (b \times \frac{1}{b}) + (a \times \frac{1}{a}) \times \frac{1}{b}}{1 - \frac{1}{a} \cdot \frac{1}{b}} \\
 &\quad (\text{Multiplicative inverse}) \\
 &= \frac{\frac{1}{a} \times \frac{b}{b} + \frac{a}{a} \times \frac{1}{b}}{1 - \frac{1}{a} \cdot \frac{1}{b}} \quad \because \frac{a}{b} = a \times \frac{1}{b} \\
 &= \frac{\frac{b}{ab} + \frac{a}{ab}}{1 - \frac{1}{ab}} \quad \because \frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd} \\
 &= \frac{b+a}{ab-1} \quad (\text{Cancellation Law})
 \end{aligned}$$

$$\begin{aligned}
 \text{iii) } \frac{\frac{a}{b} + \frac{c}{d}}{\frac{a}{b} - \frac{c}{d}} &= \frac{\frac{a}{b} \times 1 + 1 \times \frac{c}{d}}{\frac{a}{b} \times 1 - 1 \times \frac{c}{d}} \\
 &\quad (\text{multiplicative Identity})
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{\frac{a}{b} \times (d \times \frac{1}{d}) + (b \times \frac{1}{b}) \times \frac{c}{d}}{\frac{a}{b} \times (d \times \frac{1}{d}) - (b \times \frac{1}{b}) \times \frac{c}{d}} \\
 &\quad (\text{multiplicative inverse})
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{\frac{a}{b} \times \frac{d}{d} + \frac{b}{b} \times \frac{c}{d}}{\frac{a}{b} \times \frac{d}{d} - \frac{b}{b} \times \frac{c}{d}} \quad \because \frac{a}{b} = a \times \frac{1}{b} \\
 &= \frac{\frac{ad}{bd} + \frac{bc}{bd}}{\frac{ad}{bd} - \frac{bc}{bd}} \quad \because \frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{ad \times \frac{1}{bd} + bc \times \frac{1}{bd}}{ad \times \frac{1}{bd} - bc \times \frac{1}{bd}} \quad \because \frac{a}{b} = a \times \frac{1}{b}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{(ad+bc) \times \frac{1}{bd}}{(ad-bc) \times \frac{1}{bd}} \quad (\text{Dist. Law})
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{ad+bc}{ad-bc} \quad (\text{Cancellation Law})
 \end{aligned}$$

$$\begin{aligned}
 \text{iv) } \frac{\frac{1}{a} + \frac{1}{b}}{1 - \frac{1}{a} \cdot \frac{1}{b}} &= \frac{\frac{1}{a} \times 1 + 1 \times \frac{1}{b}}{1 - \frac{1}{a} \cdot \frac{1}{b}} \quad (\text{mult. Ident})
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{b \times \frac{1}{ab} + a \times \frac{1}{ab}}{1 - \frac{1}{ab}} \quad \because \frac{a}{b} = a \times \frac{1}{b} \\
 &= \frac{b \times \frac{1}{ab} + a \times \frac{1}{ab}}{ab \times \frac{1}{ab} - 1 \times \frac{1}{ab}} \\
 &\quad (\text{multiplication inverse and multi. Identity}) \\
 &= \frac{(b+a) \times \frac{1}{ab}}{(ab-1) \times \frac{1}{ab}} \quad (\text{Dist. property}) \\
 &= \frac{b+a}{ab-1} \quad (\text{Cancellation Law})
 \end{aligned}$$
